

# DRAFT: MODIFIED ZERO TIME DELAY INPUT SHAPING FOR INDUSTRIAL ROBOT WITH FLEXIBILITY

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## ABSTRACT

*Input shaping is an effective approach for vibration suppression in a variety of applications. However the time delay introduced by input shaping is not desired in some applications. Current techniques that try to reduce the time delay could not guarantee zero time delay or would cause non-smooth motion, which is harmful for the service life of actuators. In order to guarantee zero time delay, as well as the smoothness of motion, a modified zero time delay input shaping is proposed in this work. Experiment results are used to show the advantage of the proposed approach.*

## INTRODUCTION

Precision motion control is desired in a variety of industrial applications. One example of these applications, which is spot welding using industrial robot, is shown in Fig.1. Due to the flexibility in the drive train and end-effector, nonnegligible residual vibration will appear when the robot is performing a motion with high speed and/or high acceleration. Such kind of residual vibration is harmful because a) large vibration could cause collision between the robot and the workpiece, b) a robot could not perform the next task until the residual vibration stops, thus the productivity would be limited.

In order to suppress the residual vibrations without modifying the mechanical structure, advanced control techniques should be investigated. Input shaping (IS) [1, 2] is one of the most promising techniques for vibration suppression. Though input shaping is effective, easy to use, and robust to modelling error [3], the time delay introduced by input shaping is not appre-

ciated in applications with stringent requirements on operation times.

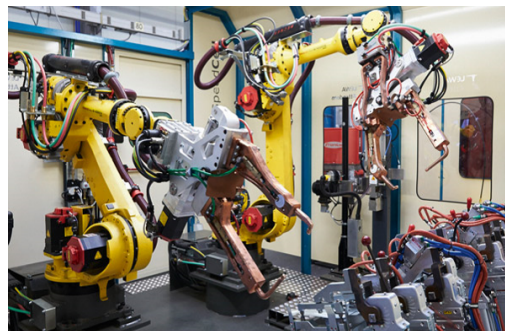


Figure 1. Industrial Robots for Spot Welding.

In order to solve the time delay issue, techniques including predictive approach [4], smith predictor [5], equal length shaper [6] and zero time delay input shaping [7] have been introduced. Some of these techniques like predictive approach or smith predictor could reduce the time delay but not totally avoid it. Equal length shaper and zero time delay input shaping, which utilize similar ideas, could totally avoid time delay of input shaping by accelerating the original input. However, the shaped motion could be non-smooth if the length of time delay and the original control input are close. The non-smooth motion must be avoided since it would cause more wear and decrease the robot service life [8, 9].

In this paper, a modified zero time delay input shaping approach is proposed to address the non-smoothness issue. The proposed approach is developed based on the zero time delay input shaping approach, thus no time delay would be introduced. Comparing to the equal length shaper or zero time delay input shaping, the proposed modification would generate more smooth motion.

This paper is organized as follows: section II reviews input shaping and zero time delay input shaping, section III shows the proposed modified zero time delay input shaping, experiment results are provided in section IV, section V concludes this work.

## REVIEW OF INPUT SHAPING

This section reviews input shaping and zero time delay input shaping. In order to reveal the connection between these two approaches, a convolution representation of input shaping is then introduced.

### Input Shaping

The idea of input shaping is: separating the input into different parts with time delay, such that the residual vibration caused by each parts would cancel each other. The structure of input shaping is represented in Fig. (2), where the 'IS' block repre-

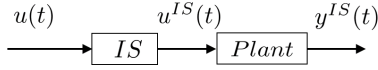


Figure 2. Input Shaping.

sents input shaping. This block divides the input  $u(t)$  into several parts with different time delay. This block is illustrated in Fig. (3).

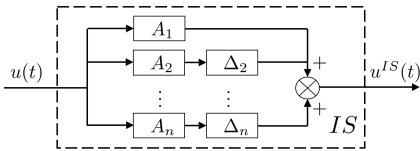


Figure 3. Time-Delay Blocks Representing Input Shaping [1].

Taking second order linear system as an example. Let the

transfer function be  $G$ ,

$$G(s) = \frac{K\omega_0^2}{s^2 + 2D\omega_0s + \omega_0^2} \quad (1)$$

where  $\omega_0$  is the natural frequency,  $D$  is the damping ratio, and  $K$  is the static gain. Let the unit impulse signal be the input, as  $u_\delta(t) = \delta(t)$ , then the output of this system, i.e. the unit impulse response is:

$$h(t) = K \frac{\omega_0}{\sqrt{1-D^2}} e^{-D\omega_0 t} \sin(\sqrt{1-D^2} \omega_0 t) \quad (2)$$

Refer to [2], let  $k = \exp\left(\frac{-D\pi}{\sqrt{1-D^2}}\right)$ ,  $A_1 = \frac{1}{1+k}$ ,  $A_2 = \frac{k}{1+k}$ ,  $\Delta_2 = \frac{\pi}{\omega_0\sqrt{1-D^2}}$ , then the shaped input  $u^{IS}(t)$  could remove residual vibration after  $\Delta_2$  since the response of  $A_1\delta(t)$  and  $A_2\delta(t - \Delta_2)$  are cancelling out each other after  $\Delta_2$ , as shown in Fig. (4).

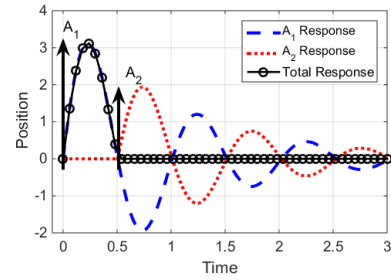


Figure 4. Vibration From Two Impulses Cancel Each Other [1].

From the aspect of convolution,  $u_\delta^{IS}(t) = (u_\delta * IS)(t)$ , where  $*$  is the convolution operator and  $IS(t)$  is a sequence of impulses, known as input shaper:

$$IS(t) = \sum_{i=1}^n A_i \delta(t - \Delta_i) \quad (3)$$

It turns out that for any input with finite length, such kind input shaper is able to effectively suppress residual vibration after delaying the original input by  $\Delta_n$ , which is the time delay of the last impulse. If the length of input  $u(t)$  is  $T$ , then the length after input shaping is  $T + \Delta_n$ . When more impulses are added, the input shaping is more robust to modelling error, but the time delay is longer [3].

## Zero Time Delay Input Shaping

The structure of zero time delay input shaping is summarized as Fig. (5). Comparing to the conventional input shaping, an 'accelerate' block is used to shorten the length of the original control input. This block makes it possible to totally eliminate the time delay.

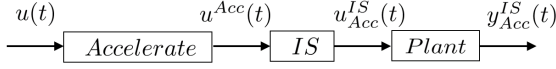


Figure 5. Zero Time Delay Input Shaping.

Let  $T$  be the length of the input  $u(t)$ , and  $\Delta_n$  be the time delay introduced by input shaping. Let a time scale parameter be  $\alpha = \frac{T - \Delta_n}{T} < 1$ , then the accelerated input is

$$u^{Acc}(t) = u\left(\frac{t}{\alpha}\right), t \in [0, \alpha T] \quad (4)$$

The length of the accelerated input is  $\alpha T$ . Input shaping is applied to the accelerated input  $u^{Acc}(t)$ , and time delay  $\Delta_n$  is added to the accelerated input. The length of the shaped input  $u^{IS}_{Acc}(t)$  is  $T$  since

$$\alpha T + \Delta_n = \frac{T - \Delta_n}{T} T + \Delta_n = T \quad (5)$$

As a result, there is no time delay compared to the original input  $u(t)$ .

## Convolution Representation of Input Shaping

Let  $h(t)$  be the impulse response of a linear system. Then the output of input  $u(t)$  is

$$y(t) = \int_0^t h(\tau) u(t - \tau) d\tau = (h * u)(t) \quad (6)$$

For conventional input shaping, the output  $y^{IS}(t)$  is

$$y^{IS}(t) = u(t) * IS(t) * h(t) = u(t) * (IS * h)(t) := u(t) * h^{IS}(t) \quad (7)$$

According to Eqn. (7), input shaping can be interpreted as modifying the impulse of dynamical system. This modification adjust impulse response from  $h(t)$  to  $h^{IS}(t) = (IS * h)(t)$ . The residual vibration can be suppressed for any input as long as the impulse response is  $h^{IS}(t)$ .

For zero time delay input shaping, the input has been accelerated to  $u^{Acc}(t)$ , and the output  $y^{IS}_{Acc}(t)$  is

$$y^{IS}_{Acc}(t) = u^{Acc}(t) * IS(t) * h(t) = u^{Acc}(t) * h^{IS}(t) \quad (8)$$

which verifies the point that the residual vibration can be suppressed if  $h^{IS}(t)$  is used.

## MODIFIED ZERO TIME DELAY INPUT SHAPING

Comparing to the conventional input shaping, zero time delay input shaping could totally eliminate the time delay introduced by input shaping. However, this approach shows drawbacks for some applications. Thus a modification of zero time delay input shaping is developed in this section.

## Drawback of Zero Time Delay Input Shaping

One necessary step in zero time delay input shaping is to accelerate the original input  $u(t)$ . When the length of the time delay and the control input are close, the accelerating would be severe, and resulting non-smooth motion due to the nature of input shaping, as shown in Fig. (6).

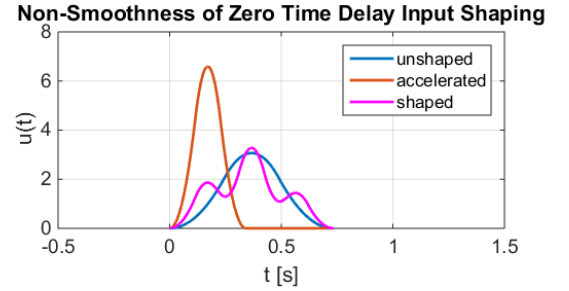


Figure 6. Drawback of Zero Time Delay Input Shaping: Close Time Length and Time Delay Result in Non-Smoothness.

In Fig. (6), the length of  $u(t)$  is close to  $\Delta_n$ , thus the time scale  $\alpha$  is close to 0 and a severe accelerating is performing on the input  $u(t)$ . After input shaping, the accelerated input is separated, resulting several peaks in the shaped input. Such input is less smooth comparing to the original input. If the non-smooth input is used, the changing rate of input could exceed the actuator's limit and cause more wear in the actuator's mechanical parts.

## Modified Zero Time Delay Input Shaping

In order to the non-smoothness issue, a modified zero time delay input shaping is proposed. The structure of the proposed approach is summarized as Fig. (7).

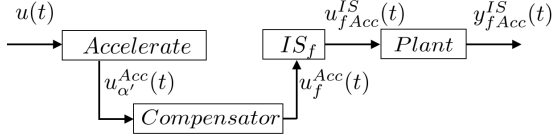


Figure 7. Modified Zero Time Delay Input Shaping.

Comparing to zero time delay input shaping, the modification here is a compensator block. The function of this block is to reduce time delay required for input shaping, thus no severe accelerating is required for the input  $u(t)$ .

This section presents the design of this compensator and corresponding input shaper  $IS_f$  from the aspect of convolution product.

**Design of Modified Zero Time Delay Input Shaping** In zero time delay input shaping,  $u(t)$  is accelerated by time scale  $\alpha$ , but  $h^{IS}(t) = (IS * h)(t)$  is not scaled at all. In this paper, we consider the idea that accelerating  $u(t)$  and  $h^{IS}(t)$  by the same time scale  $\alpha'$ , such that

$$u_{\alpha'}^{Acc}(t) = u\left(\frac{t}{\alpha'}\right) \quad (9)$$

and  $h_{\alpha'}^{IS}(t) = h^{IS}\left(\frac{t}{\alpha'}\right)$ . According to the time scaling property of convolution [10], let  $y_{\alpha'}^{IS}(t) = \frac{1}{\alpha'} h_{\alpha'}^{IS}(t) * u_{\alpha'}^{Acc}(t)$ , then  $\forall t \in [0, \alpha' T]$

$$y_{\alpha'}^{IS}(t) = \frac{1}{\alpha'} \int_0^t u\left(\frac{\tau}{\alpha'}\right) h^{IS}\left(\frac{t}{\alpha'} - \frac{\tau}{\alpha'}\right) d\tau = y^{IS}\left(\frac{t}{\alpha'}\right) \quad (10)$$

Let  $T$  be the length of input, and  $\Delta_n$  be the dime delay of input shaping. In order to guarantee zero time delay, there should be  $\alpha'(T + \Delta_n) = T$ , thus

$$\alpha' = \frac{T}{T + \Delta_n} > \frac{T - \Delta_n}{T} = \alpha \quad (11)$$

where  $\alpha$  is the time scale of zero time delay input shaping from Eqn. (5). Since  $\alpha'$  is larger, the accelerating is less severe than zero time delay input shaping. The function of the compensator and corresponding input shaper is to adjust  $h^{IS}(t)$  to  $\frac{1}{\alpha'} h_{\alpha'}^{IS}(t)$ .

**Compensator** The design of the compensator then became a problem to find a signal  $f(t)$ , such that the impulse response  $h(t)$  can be accelerated and scaled to  $\frac{1}{\alpha'} h_{\alpha'}(t)$ ,

$$h_{\alpha'}(t) = f(t) * h(t) = \frac{1}{\alpha'} h\left(\frac{t}{\alpha'}\right) \quad (12)$$

The compensator can be designed in either frequency domain or time domain, as long as the impulse response of the designed filter is  $f(t)$ . A frequency domain design example will be given later.

**Input Shaper** The input shaper can be designed as:

$$IS_f(t) = \sum_{i=1}^n A_i \delta(t - \alpha' \Delta_i) \quad (13)$$

where  $A_i$  and  $\Delta_i$  are the same as in Eqn. (3).

Since

$$h^{IS}(t) = (IS * h)(t) = \sum_{i=1}^n A_i h(t - \Delta_i) \quad (14)$$

therefore

$$(h_{\alpha'} * IS_f)(t) = \sum_{i=1}^n A_i \frac{h\left(\frac{t}{\alpha'} - \Delta_i\right)}{\alpha'} = \frac{h^{IS}\left(\frac{t}{\alpha'}\right)}{\alpha'} = \frac{1}{\alpha'} h_{\alpha'}^{IS}(t) \quad (15)$$

Thus the designed input shaper and compensator can adjust  $h^{IS}(t)$  to  $\frac{1}{\alpha'} h_{\alpha'}^{IS}(t)$ .

**Example on a Second Order Linear System** Suppose the plant is a second order linear system with transfer function  $G$  as in Eqn. (1). The impulse response of the system is shown in Eqn. (2). The transfer function of another system which has impulse response  $\frac{1}{\alpha'} h\left(\frac{t}{\alpha'}\right)$  is:

$$G^{Acc}(s) = \frac{K\omega_0^2}{(\alpha's)^2 + 2\alpha'D\omega_0s + \omega_0^2} \quad (16)$$

Then the transfer function  $F(s)$  of the compensator can be designed as

$$F(s) = \frac{G^{Acc}(s)}{G(s)} = \frac{s^2 + 2D\omega_0s + \omega_0^2}{(\alpha's)^2 + 2\alpha'D\omega_0s + \omega_0^2} \quad (17)$$

which is causal. According to the property of Laplace transform [11],

$$G^{Acc}(s) = F(s)G(s) \Rightarrow \frac{1}{\alpha'} h\left(\frac{t}{\alpha'}\right) = f(t) * h(t) \quad (18)$$

Thus the impulse response of this compensator  $f(t)$  agrees with Eqn. (12).

The input shaper can be firstly designed for  $G(s)$  using any input shaping design technique. Then the input shaper for modified zero time delay input shaping can be designed as Eqn. (13), i.e. scale time delay for each impulse.

## EXPERIMENT RESULT

The proposed approach has been tested on FANUC M-16iB industrial robot with an experimental flexible payload as shown in Fig. (8). The flexible payload is designed to have similar natural frequency as a large end-effector of industrial robot. A wireless accelerometer is attached at the end tip of the payload for monitoring residual vibration.

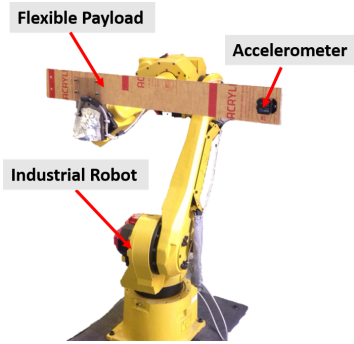


Figure 8. Robot with flexible payload.

In the experiment, the robot is performing a rapid rest-to-rest motion along  $X$  direction in the workspace. The motion path is illustrated in Fig. (9). The position, velocity, and acceleration reference along  $X$  direction are shown in Fig. (10).

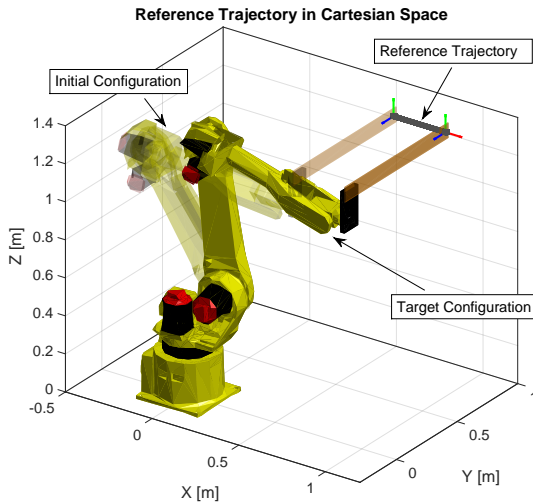


Figure 9. Reference trajectory of robot in Cartesian space.

According to the acceleration measured by the wireless accelerometer, the flexible payload can be approximately identified

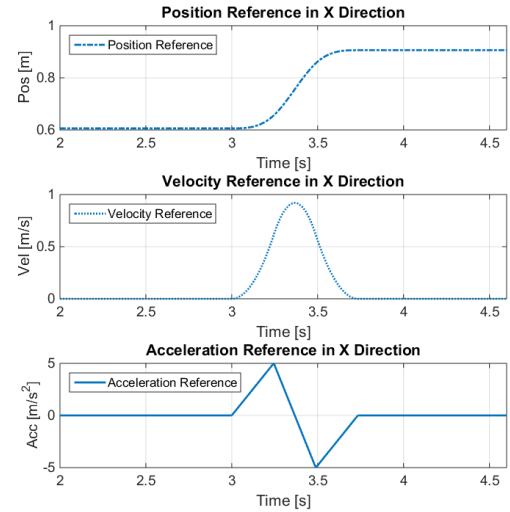


Figure 10. Position, velocity, and acceleration reference along  $X$  direction.

as a second order linear system. Fig. (11) shows the measured acceleration and the estimated acceleration from the identified model. The identified natural frequency of the system is 2.55Hz, and the damping ratio is 0.04.

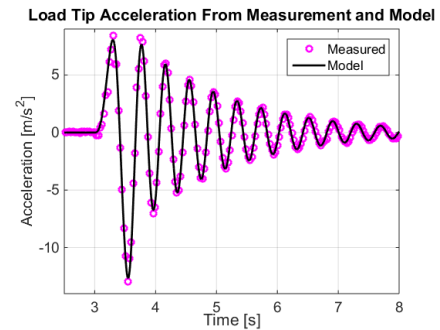


Figure 11. Measured and estimated acceleration at the end tip of payload.

Input shaping, zero time delay input shaping, and modified zero time delay input shaping are tested in the experiment. All of the three approaches use Zero Vibration and Derivative (ZVD) shaper as  $IS(t)$  for robustness consideration [2]. Fig. (12) shows the measured payload tip acceleration for the three approaches. The desired acceleration is also shown as reference.

As shown in Fig. (12), the residual vibration can be effectively suppressed by all of the approaches. The proposed ap-

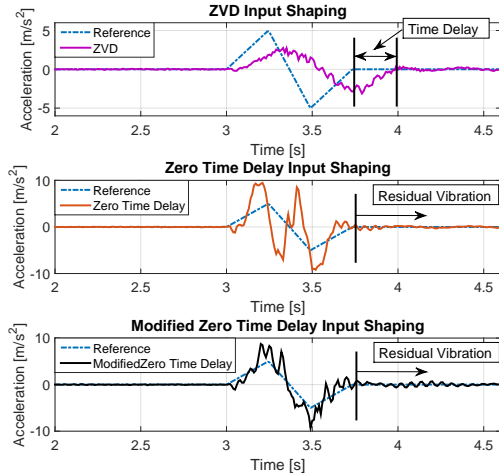


Figure 12. Experiment results.

proach and zero time delay input shaping introduce no time delay while input shaping introduces a time delay which adds about 40% time to the original motion. The proposed approach shows smoother acceleration than zero time delay input shaping.

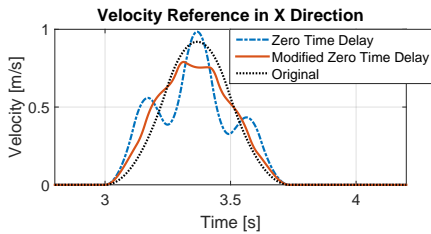


Figure 13. Comparison of velocity reference in X direction.

In the experiment,  $\alpha' = 0.6516$ , and  $\alpha = 0.4653$ . Such time scaling factors mean that in zero time delay input shaping, the motion reference has been shortened to around 47% of the original length, while it is only 65% for the proposed approach. Since the modified zero time delay input shaping avoids severe accelerating, smoother robot motion can be used to suppress the vibration. Fig. (13) plots the velocity reference in X direction of the proposed approach and zero time delay input shaping to show the smoothness comparison result more clearly.

## CONCLUSION

In this paper, a modified zero time delay input shaping approach has been proposed for vibration suppression of industrial

robot with flexibility. Comparing to existing input shaping techniques, the proposed approach can fully compensate the time delay introduced by conventional input shaping. Furthermore, the proposed approach produces smoother motion than current techniques that could avoid time delay. Experiment result has shown the proposed approach outperforms conventional input shaping and zero time delay input shaping in terms of time delay and motion smoothness.

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