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Mobility Analysis of a Sarrus Linkage-like 7-R Single Closed Loop Mechanism*

Yu Zhao, Tiemin Li, Xiaowen Yu, Xiaoqiang Tang, and Liping Wang

Abstract— **This paper introduces the mobility analysis of a 1-DOF (Degrees of Freedom) Sarrus Linkage like 7-R single closed loop mechanism. The analysis procedure implements both group theory based criterion and reciprocal screw theory. The analysis shows that such mechanism allows rectilinear motion with full-cycle mobility. Further, unlike the Sarrus Linkage, which is overconstrained, this 7-R linkage is non-overconstrained. This property makes the Sarrus Linkage-like mechanism more robust to manufacturing errors. Simulation has shown that even with large manufacturing errors, such mechanism is still able to realize desired rectilinear motion. One possible application for this mechanism is a durable automotive suspension.**

I. Introduction

Mobility analysis for mechanisms, as the foundation for mechanism analysis and synthesis, has a long history. The most famous contribution is obviously the Kutzbach-Grübler criterion. This criterion is easy to use and it is possible to implement in many applications. However, as reported by J. M. Hervé, this simple criterion can only be applied to one class of mechanisms called "trivial" linkages [1]. There exist lots of examples that real mobility of linkages does not correspond to the computation result of Kutzbach-Grübler criterion. In fact, it is well known that such criterion does not apply to overconstrained linkages. The first example of such linkages that reported in literature is Sarrus Linkage [2]. J. M. Hervé has made pioneering contributions in applying group theory to study mechanisms. According his work, spatial linkages can be classified as "trivial", "exceptional" and "paradoxical" linkages. P. Fanghella and C. Galletti have extended J. M. Herve's work to develop an algorithmic approach [3, 4, 5]. J.M. Rico and B. Ravani have shown that using group theory one can distinguish between the three different kinds of linkages and they concluded a mobility criterion applicable to all exceptional chains and trivial chains [6, 7]. On the other hand, K. H. Hunt has provided a different method that employing results from infinitesimal kinematics to list a collection of screw systems which allow "full-cycle mobility" [8, 9, 10]. J. M. Selig shows that these screw systems are subalgebras of the Euclidean group's Lie algebra [11]. Z. Huang has presented another mobility analysis procedure using reciprocal screw theory [12, 13].

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Based on these work, it is not difficult to recognize Sarrus Linkage as an exceptional linkage with full-cycle mobility. Because the intersection of the two subgroups which are generated by the clockwise chain and the counterclockwise chain of Sarrus Linkage is a straight line, this mechanism can be called a rectilinear mechanism. Inspired by the rectilinear mechanism synthesis work of J.S. Zhao, we simply add one revolute joint to the Sarrus Linkage [14]. As a result, the 7-R single closed loop kinematic chain is a trivial linkage. This mechanism allows full-cycle mobility. Using reciprocal screw theory, we have found this 7-R mechanism can still perform rectilinear motion. Further, we have found that the rectilinear motion is robust to manufacturing errors.

This paper is organized as follows: section 2 reviews mobility analysis for Sarrus Linkage, performs mobility analysis for this mechanism and compares the two results; section 3 analyze mobility of this mechanism with 3 classes of manufacturing error and indicates that with these kinds of manufacturing error, the mechanism can still perform rectilinear motion; section 4 shows the motion simulation result with and without manufacturing error; section 6 made the conclusion for this paper.

II. Mobility Analysis for Sarrus Linkage and Sauurs Linkage-like 7-R Single Closed Loop Mechanism

A. Mobility Analysis for Sarrus Linkage

Figure 1 gives an example of Sarrus Linkage. Sarrus Linkage transfers circular motion to linear motion.

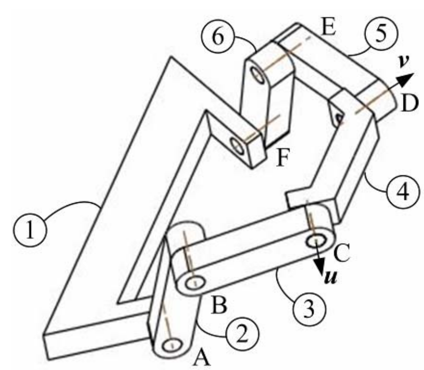

Figure 1. An example of Sarrus Linkage.

The four links 1, 2, 3 and 4 are connected by three revolute joints A, B, and C that parallels to each other, as the links 1, 5, 6 and 4 that connected by three revolute joints D, E and F. The direction of the two groups of revolute joints are different. The mobility of Sarrus Linkage has been analyzed in [6] as an example of exceptional chains. The first three revolute joints A, B and C generate a planar motion subgroup P_u with a normal vector, $\boldsymbol{u}$, parallel to the direction of joint A.

Similarly, the last three revolute joints D, E and F generate another planar motion subgroup P_v with a normal vector, $\boldsymbol{v}$, parallel to the direction of joint D. As mentioned above, joint A and joint D have different directions, thus the normal vector of the two planes are linearly independent. When considering the relative displacements of link 4 with respect to link 1, the result is the intersection of the two planar motion subgroups, P_v and P_v. The intersection, as mentioned in [6], is another subgroup $T_{u\times v}$, which stands for translations along the direction that normal to both the $\boldsymbol{u}$ and $\boldsymbol{v}$. Applying mobility criterion proposed by Rico and Ravani, the mobility of Sarrus Linkage is

$$F=\sum_{i=1}^{6} f_i-\dim\left(P_u\right)-\dim\left(P_v\right)+\dim\left(T_{u\times v}\right). \quad (1)$$

In which F is the number of DOF of the kinematic chain, f_i is the number of DOF of the i_{th} joint, dim is the dimension of the respective subgroup. Obviously, the computational result is 6-3-3+1=1.

B. *Mobility Analysis for Sarrus Linkage-like 7-R Single Closed Loop Mechanism*

Figure 2 shows the structure of the 7-R single closed loop mechanism.

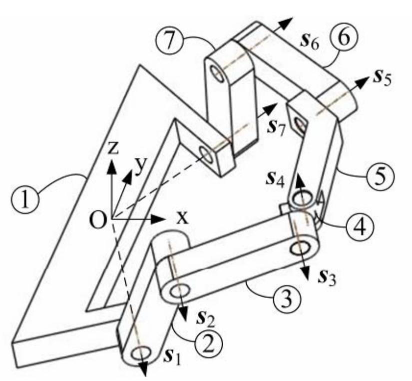

Figure 2. 7-R single closed loop mechanism.

The direction of the first three revolute joints $\boldsymbol{s}_1$, $\boldsymbol{s}_2$ and $\boldsymbol{s}_3$ are the same. The direction of the last three revolute joints $\boldsymbol{s}_5$, $\boldsymbol{s}_6$ and $\boldsymbol{s}_7$ are the same and are different from the first three revolute joints. The axis direction of revolute joint 4 is linear independent to that of $\boldsymbol{s}_1$ and $\boldsymbol{s}_7$. Link 1 is chosen to be the fixed frame and link 5 is chosen as moving platform of end effecter of this mechanism.

Attaching coordinate system to this kinematic chain. We assume that the first revolute joint $\boldsymbol{s}_1$ and last joint $\boldsymbol{s}_7$ are coplanar. The origin of the reference coordinate system is at the intersection of $\boldsymbol{s}_1$ and $\boldsymbol{s}_7$, as shown in Fig. 2. Let $\boldsymbol{r}_i$ be the position of a point at the i_{th} joint axis. Let a) $\boldsymbol{r}_1=\boldsymbol{r}_7=0$, and b) $\boldsymbol{r}_2$ and $\boldsymbol{r}_3$ are not collinear, c) $\boldsymbol{r}_5$ and $\boldsymbol{r}_6$ are not collinear, d) $\boldsymbol{s}_1(\boldsymbol{r}_2-\boldsymbol{r}_3)=0$ and $\boldsymbol{s}_7(\boldsymbol{r}_4-\boldsymbol{r}_5)=0$. In addition, let

$$\begin{cases}\boldsymbol{s}_1\notin\operatorname{span}\left\{\boldsymbol{r}_2,\boldsymbol{r}_3\right\}\\ \boldsymbol{s}_7\notin\operatorname{span}\left\{\boldsymbol{r}_5,\boldsymbol{r}_6\right\}\\ \left(\boldsymbol{r}_5-\boldsymbol{r}_6\right)\cdot\left(\boldsymbol{s}_1\times\boldsymbol{s}_7\right)\neq 0\end{cases}. \quad (2)$$

These condition means that $\boldsymbol{s}_1$ is not in the plane that cross $\boldsymbol{r}_2$ and $\boldsymbol{r}_3$, $\boldsymbol{s}_7$ is not in the plane that cross $\boldsymbol{r}_5$ and $\boldsymbol{r}_6$, $\boldsymbol{r}_5$-$\boldsymbol{r}_6$ is not in the plane that cross $\boldsymbol{s}_1$ and $\boldsymbol{s}_7$. Otherwise, this mechanism will suffer singularity. Considering the linearly dependence of screw of the first 6 joint. If they are linearly dependent, there must exist a group or real numbers a_1, a_2... a_6 that are not all 0, makes

$$\sum_{i=1}^{6} a_i\boldsymbol{\xi}_i=\sum_{i=1}^{6} a_i\begin{bmatrix}\boldsymbol{s}_i\\ \boldsymbol{r}_i\times\boldsymbol{s}_i\end{bmatrix}=0. \quad (3)$$

In which $\boldsymbol{\xi}_\text{i}$ denotes screw or twist of the i_{th} joint. Because $\boldsymbol{s}_1$, $\boldsymbol{s}_4$ and $\boldsymbol{s}_7$ are linear independent, and $\boldsymbol{s}_1=\boldsymbol{s}_2=\boldsymbol{s}_3$, $\boldsymbol{s}_5=\boldsymbol{s}_6=\boldsymbol{s}_7$, Eq. 3 derives that $a_1+a_2+a_3=0$, $a_4=0$, $a_5+a_6=0$ and

$$\left(a_2\boldsymbol{r}_2+a_3\boldsymbol{r}_3\right)\times\boldsymbol{s}_1+\left(a_5\boldsymbol{r}_5+a_6\boldsymbol{r}_6\right)\times\boldsymbol{s}_7=0. \quad (4)$$

Calculating inner product with $\boldsymbol{s}_1$ of both side,

$$a_5\left(-\boldsymbol{s}_1\times\boldsymbol{s}_7\right)\cdot\left(\boldsymbol{r}_5-\boldsymbol{r}_6\right)=0. \quad (5)$$

Ref to Eq. 2 and Eq. 5, we can derivate $a_5=0$. Therefore $(a_2\boldsymbol{r}_2+a_3\boldsymbol{r}_3)\times\boldsymbol{s}_1=0$, which means there exists k that

$$a_2\boldsymbol{r}_2+a_3\boldsymbol{r}_3=k\boldsymbol{s}_1. \quad (6)$$

If k is not 0, then $\boldsymbol{s}_1=a_2/k\ \boldsymbol{r}_2+a_3/k\ \boldsymbol{r}_3\in\operatorname{span}\{\boldsymbol{r}_2, \boldsymbol{r}_3\}$. This is contradict to Eq. 2. Thus k must be 0, which means $a_2\boldsymbol{r}_2+a_3\boldsymbol{r}_3=0$. This is contradict to the assumption that $\boldsymbol{r}_2$, $\boldsymbol{r}_3$ not collinear. Thus $a_2=a_3=0$ and $a_1=a_2=...=a_6=0$. Therefore the first 6 twists $\boldsymbol{\xi}_1$, $\boldsymbol{\xi}_2$, ... $\boldsymbol{\xi}_6$ must be linear independent. Thus span$\{\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, ... \boldsymbol{\xi}_6\}$ is a 6 dimension linear vector space, and also the Lie algebra, $e(3)$, of the Euclidean group, $E(3)$.

We use mobility criterion proposed by J. M. Rico and B. Ravani [7]. Considering the counterclockwise mechanical chain connecting link 1 and link 7, the counterclockwise infinitesimal mechanical liaison is $V_{cc}(1,7)=e(3)$, thus the counterclockwise closure subalgebra is $A_{cc}=V_{cc}=e(3)$. Considering the clockwise mechanical chain connecting link 1 and link 7, then $V_c(1,7)=\operatorname{span}\{\boldsymbol{\xi}_7\}=r$, which makes $A_c(1,7)=V_c(1,7)=r$. The absolute closure subalgebra, $A_a(1,7)$, i.e. the intersection of A_{cc} and A_c, is A_c itself. Therefore we have

$$A_a(1,7) = A_c(1,7) < A_{cc}(1,7) = e(3) . \quad (7)$$

Then, the kinematic chain is movable and its mobility is

$$F = \sum f_i - \dim(e(3)) = 7 - 6 = 1 . \quad (8)$$

The mechanism can be classified as trivial kinematic chain. This procedure ensures the mechanism has full-cycle mobility. However, it is not obvious that the relative displacement of link 5 with respective to link 1 can not be recognized as translation as the open kinematic chain with the first four revolute joint is not a mechanical generator of any Lie sub group. This makes a huge difference between this 7-R single closed loop mechanism and the Sarrus Linkage. That is the reason we adopts reciprocal screw theory based mobility analysis method as follows.

According to analysis of the linear dependence of the first six twists, it is safe to say the first four twists are linear independent now. Any twist $\xi_r=[\boldsymbol{s};\boldsymbol{s}_0]$ of the reciprocal screw system of the first four twist must satisfy

$$\boldsymbol{s}\cdot(\boldsymbol{r}_i \times \boldsymbol{s}_i) + \boldsymbol{s}_0 \cdot \boldsymbol{s}_i = 0, i = 1,2,3,4 . \quad (9)$$

Let $\boldsymbol{s}_r=\boldsymbol{s}_1\times\boldsymbol{s}_4$. It is not difficult to verify that the following two twists fulfill the requirement of Eq. 9:

$$\begin{cases} \xi_{r1} = [0 \quad ; \quad \boldsymbol{s}_r], \\ \xi_{r2} = [\boldsymbol{s}_1 \quad ; \quad \boldsymbol{r}_4 \times \boldsymbol{s}_1]. \end{cases} \quad (10)$$

Obviously the two twists are linear independent. Thus the reciprocal screw system R_{cc} = span{ξ_{r1}, ξ_{r2}}. Let $\boldsymbol{s}_{rr}=\boldsymbol{s}_7\times\boldsymbol{s}_4$. Similarly, the reciprocal screw system of the last three twists contains three linear independent twists as following:

$$\begin{cases} \xi_{rr1} = [\boldsymbol{s}_7 \quad ; \quad 0], \\ \xi_{rr2} = [0 \quad ; \quad \boldsymbol{s}_{rr}], \\ \xi_{rr3} = [0 \quad ; \quad \boldsymbol{s}_1 \times \boldsymbol{s}_7]. \end{cases} \quad (11)$$

Thus the reciprocal screw system of the last three twists is R_c = span{ξ_{rr1}, ξ_{rr2}, ξ_{rr3}}. The reciprocal screw system of link 5 is $R_l = R_{cc}+R_c$, which stands for the constraint force of link 5. The reciprocal screw system of the reciprocal screw system, which stands for the motion under the constraint, contains linear independent twists as following:

$$\xi_m = [0 \quad ; \quad \boldsymbol{s}_1 \times \boldsymbol{s}_7]. \quad (12)$$

The infinite small motion of link 5 is M_l = span{ξ_m}. The physical meaning of this twist is a translation along $\boldsymbol{s}_1\times\boldsymbol{s}_7$. As $\boldsymbol{s}_1$ and $\boldsymbol{s}_7$ are fixed vector in space, the relative motion of link 5 with respective to link 1 is always a translation along $\boldsymbol{s}_1\times\boldsymbol{s}_7$.

Based on the mobility analysis based on group theory and reciprocal screw theory, we conclude that this 7-R single closed loop mechanism allows full-cycle rectilinear motion. Noting that Sarrus Linkage allows translation along the same direction, i.e., they have the same mobility.

III. Mobility Analysis with Existence of Manufacturing Errors

This paper considers 3 class of manufacturing error: (1) link length error, (2) frame angle error, and (3) end effecter angle error, as shown in Fig. 3.

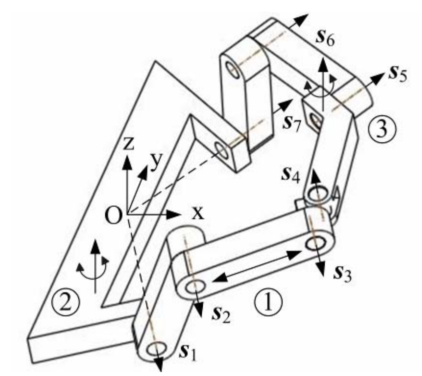

Figure 3. Three classes of manufacturing errors.

Assume that manufacturing error is very small, then we can consider them as "error twist". Manufacturing error of class 1 does not affect the mobility of a Sarrus Linkage, as well as the 7-R single closed loop mechanism. Using similar coordinate system and symbols to denote joint directions and joint axis positions as last section. Let ξ_{e1} = [0; $\boldsymbol{r}_{e1}$] be the link length error. Assume that $\Delta(\boldsymbol{r}_2-\boldsymbol{r}_3)=\boldsymbol{r}_{e1}$, $\boldsymbol{r}_1$=0, and $\boldsymbol{s}_1(\boldsymbol{r}_2-\boldsymbol{r}_3)$=0, we need to find 3 numbers a_1, a_2 and a_3 that

$$a_1\begin{bmatrix}\boldsymbol{s}_1\\0\end{bmatrix} + a_2\begin{bmatrix}\boldsymbol{s}_1\\\boldsymbol{r}_2\times\boldsymbol{s}_1\end{bmatrix} + a_3\begin{bmatrix}\boldsymbol{s}_1\\\boldsymbol{r}_3\times\boldsymbol{s}_1\end{bmatrix} = \begin{bmatrix}0\\\boldsymbol{r}_{e1}\end{bmatrix}. \quad (13)$$

Equation 15 requires that

$$\begin{cases} a_2\boldsymbol{r}_3\cdot(\boldsymbol{r}_2\times\boldsymbol{s}_1) = dl(\boldsymbol{r}_2-\boldsymbol{r}_3)\cdot\boldsymbol{r}_3 , \\ a_3\boldsymbol{r}_2\cdot(\boldsymbol{r}_3\times\boldsymbol{s}_1) = dl(\boldsymbol{r}_2-\boldsymbol{r}_3)\cdot\boldsymbol{r}_2 . \end{cases} \quad (14)$$

Because $\boldsymbol{r}_2$ and $\boldsymbol{r}_3$ intersect at origin, they are not parallel. Thus $(\boldsymbol{r}_2-\boldsymbol{r}_3)\boldsymbol{r}_3$ and $(\boldsymbol{r}_2-\boldsymbol{r}_3)\boldsymbol{r}_2$ can not be zero at the same time. This means we can find a_1, a_2 and a_3 that not all 0, i.e.

$$\xi_{e1} \in \text{span}\{\xi_1, \xi_2, \xi_3\} . \quad (15)$$

Exponential mapping maps infinite small screw to finite displacement. If there is no manufacturing error, Sarrus Linkage is movable if there exists joint displacements $\theta_{cc}\boldsymbol{\xi}_{cc}$

$$e^{\theta_{cc}\xi_{cc}} = e^{-\theta_3\xi_3} e^{-\theta_2\xi_2} e^{-\theta_1\xi_1} = e^{\theta_7\xi_7} e^{\theta_6\xi_6} e^{\theta_5\xi_5} = e^{\theta_c\xi_{cc}} . \tag{16}$$

In which $\theta_{cc}\boldsymbol{\xi}_{cc} = \theta_c\boldsymbol{\xi}_c \in A_a(1,4)$. This condition will be modified when there exists link length error

$$e^{\theta_{cc}\xi_{cc}} = e^{-\tilde{\theta}_3\xi_3} e^{-\xi_{e1}} e^{-\tilde{\theta}_2\xi_2} e^{-\tilde{\theta}_1\xi_1} = e^{\theta_7\xi_7} e^{\theta_6\xi_6} e^{\theta_5\xi_5} = e^{\theta_c\xi_{cc}} . \tag{17}$$

As $\theta_{cc}\boldsymbol{\xi}_{cc} \in A_{cc}(1,4) = \text{span}\{\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \boldsymbol{\xi}_3\}$, $\boldsymbol{\xi}_{e1} \in \text{span}\{\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \boldsymbol{\xi}_3\} = A_{cc}(1,4)$, we can derive that

$$\begin{cases} \theta_{cc}\boldsymbol{\xi}_{cc} = k_1\boldsymbol{\xi}_1 + k_2\boldsymbol{\xi}_2 + k_3\boldsymbol{\xi}_3 \ , \\ e^{-\tilde{\theta}_3\xi_3} e^{-\xi_{e1}} e^{-\tilde{\theta}_2\xi_2} e^{-\tilde{\theta}_1\xi_1} = e^{n_1\xi_1 + n_2\xi_2 + n_3\xi_3} \ . \end{cases} \tag{18}$$

Let $k_1=n_1$, $k_2=n_2$, $k_3=n_3$, Eq. 17 is satisfied. Thus the existence of $\theta_{cc}\boldsymbol{\xi}_{cc}$ can be guaranteed, i.e. we can choose different joint positions to make the mechanism movable.

This is because the link length error can be compensated by the first three joints. Similar analysis can be taken on the 7-R single closed loop mechanism to prove that the link length error will not affect the mobility of 7-R mechanism either. It does not matter which link has error.

However, angle errors will affect the mobility of Sarrus Linkage. This is obvious in Fig. 4. If there are angle errors, the sum of the interior angles in the projective quadrilateral will not maintain 360 degrees. Thus Sarrus Linkage can not be assembled. The reason that actual Sarrus Linkage can move is the joint clearance and link deformation.

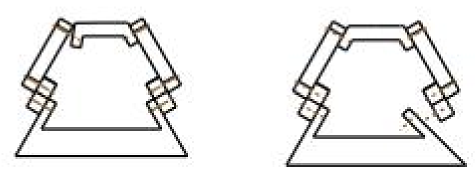

Figure 4. Effect of (a) end effecter angle error, (b) frame angle error.

For the 7-R closed loop mechanism, angle errors will not affect the mobility. The condition for the 7-R closed loop mechanism to be movable is the existence of $\theta_{cc}\boldsymbol{\xi}_{cc} = \theta_c\boldsymbol{\xi}_c \in A_a(1,7)$, that

$$e^{\theta_{cc}\xi_{cc}} = e^{-\theta_6\xi_6} e^{-\theta_5\xi_5} e^{-\theta_4\xi_4} e^{-\theta_3\xi_3} e^{-\theta_2\xi_2} e^{-\theta_1\xi_1} = e^{\theta_7\xi_7} = e^{\theta_c\xi_{cc}} . \tag{19}$$

Similarly, angle errors can be represented with two error twists: $\boldsymbol{\xi}_{e2} = [\boldsymbol{s}_{e2}; \boldsymbol{r}_{e2}\times\boldsymbol{s}_{e2}]$ and $\boldsymbol{\xi}_{e3} = [\boldsymbol{s}_{e3}; \boldsymbol{r}_{e3}\times\boldsymbol{s}_{e3}]$. Because $A_{cc}(1,7)=V_{cc}(1,7)=e(3)$, $\boldsymbol{\xi}_{e2}$, $\boldsymbol{\xi}_{e3}$, $\boldsymbol{\xi}_7 \in e(3)$, we can derive that

$$\begin{cases} e^{\xi_{e2}} e^{\theta_7\xi_7} e^{\xi_{e3}} = e^{k_1\xi_1 + k_2\xi_2 + k_3\xi_3 + k_4\xi_4 + k_5\xi_5 + k_6\xi_6} \ , \\ e^{-\tilde{\theta}_1\xi_1} e^{-\tilde{\theta}_2\xi_2} e^{-\tilde{\theta}_3\xi_3} e^{-\tilde{\theta}_4\xi_4} e^{-\tilde{\theta}_5\xi_5} e^{-\tilde{\theta}_6\xi_6} = e^{n_1\xi_1 + n_2\xi_2 + n_3\xi_3 + n_4\xi_4 + n_5\xi_5 + n_6\xi_6} \ . \end{cases} \tag{20}$$

Let $n_i=k_i$, i=1...6. Then we have

$$e^{\theta_{cc}\xi_{cc}} = e^{-\xi_{e2}} e^{-\tilde{\theta}_6\xi_6} e^{-\tilde{\theta}_5\xi_5} e^{-\tilde{\theta}_4\xi_4} e^{-\tilde{\theta}_3\xi_3} e^{-\tilde{\theta}_2\xi_2} e^{-\tilde{\theta}_1\xi_1} e^{-\xi_{e3}} = e^{\theta_7\xi_7} = e^{\theta_c\xi_{cc}} . \tag{21}$$

Therefore the 7-R closed loop mechanism is movable, i.e. the manufacturing error can be compensated by the movements of joints. Though error of class 2 and 3 will cause small changes of joint direction and position, since mobility analysis based on reciprocal theory does not require special condition of joint directions and axis positions, thus as long as these small changes do not contradict Eq. 2, with the same choice of joint axis position $\boldsymbol{r}_i$, mobility analysis will give the same results. The mechanism with angle errors is shown in Fig. 5.

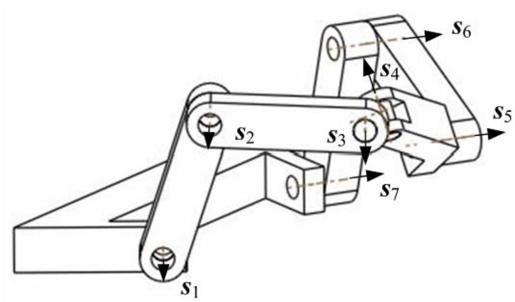

Figure 5. 7-R single closed loop mechanism with manufacturing error.

Based on the mobility analysis with three classes of manufacturing error, we can conclude that even with manufacturing errors, this mechanism can still perform the same rectilinear motion as Sarrus Linkage. The rationale of this property can be regarded as the infinite small motions generated by the counterclockwise open chain is the relative complement set of the infinite small motions generated by the clockwise open chain in the Lie algebra, $e(3)$, of the 3-dimentional Euclidean space, $E(3)$. Thus error compensation is enabled. This rationale also implies the extra revolute joint can not be placed arbitrarily.

IV. Motion Simulation

Motion simulation is performed with CAD software Pro/Engineer. The mechanism is firstly modeled without manufacturing errors, then with the three kinds of errors mentioned in section III. The three classes of manufacturing errors are introduced to the simulation respectively first, and then together.

The mechanism model refers to Fig. 2. We assume that link 2, 3, 6, 7 have identical lengths. Joint 7($\boldsymbol{s}_7$) is chosen as the actuator. The positions of all other joints during the motion simulation are shown in Fig. 6.

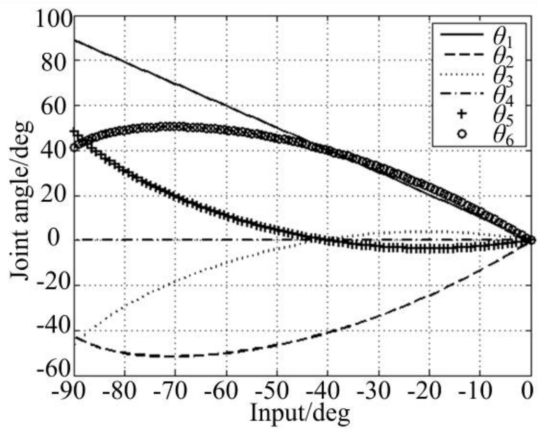

Figure 6. Positions of joint 1, 2, ... 6.

The simulation shows that translations along x and y axis and rotations about x, y, z axis do not exist. Let translation along z axis be the output of this mechanism, the input-output curve is shown in Fig. 7.

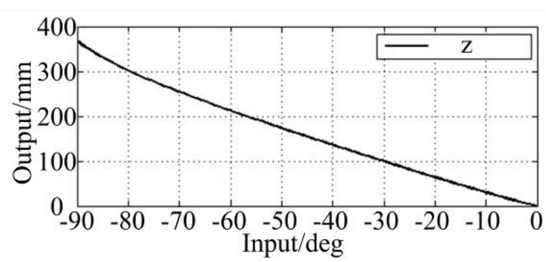

Figure 7. Input-output curve of 7-R single closed loop mechanism.

If there is only link length error, we assume link 2 has a link length error up to 20% of its original length. Motion simulation shows that link 5 can only move along z axis and the input-output curve is shown in Fig. 8.

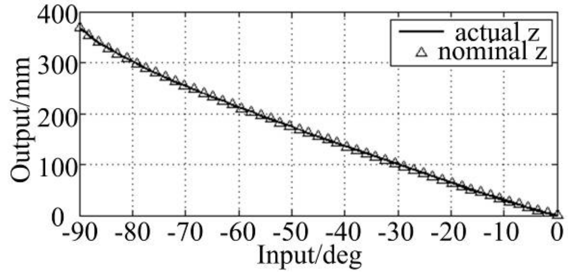

Figure 8. Input-output curve with link length error.

If there is only frame angle error, we assume the angle error is up to 10 degrees. The motion simulation shows that link 5 can only move along z axis and the input-output curve is shown in Fig. 9.

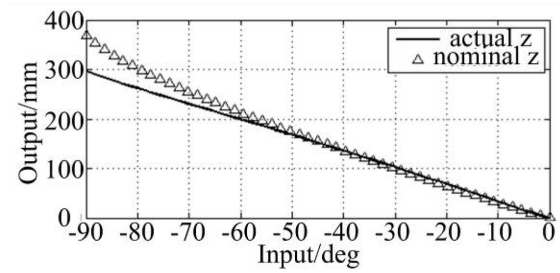

Figure 9. Input-output curve with frame angle error.

If there is only end effecter angle error, we assume the angle error is up to 10 degrees. The motion simulation shows that link 5 can only move along z axis and the input-output curve is shown in Fig. 10.

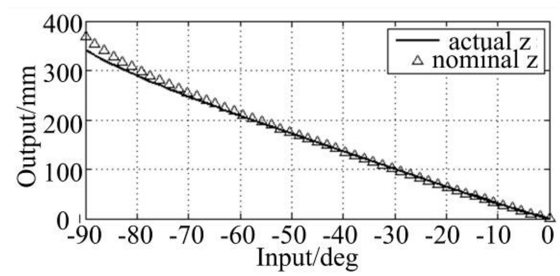

Figure 10. Input-output curve with end effecter angle error.

If link length error and angle error occur at the same time, we assume the link length error, the frame assembling angle error and end effecter manufacturing angle error are the same errors in the simulations above. The simulation shows that link 5 can only move along z axis. The input-output curve is shown in Fig. 11.

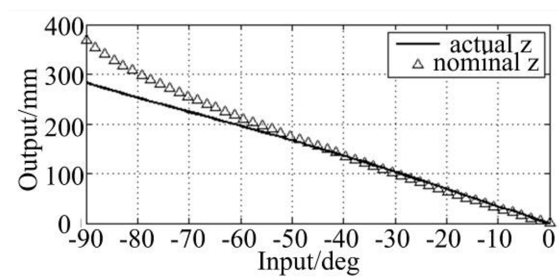

Figure 11. Input-output curve with manufacturing errors.

It can be seen that the link length error, has very little affect to the rectilinear motion of link 5; the frame angle error, has the most significant affect to the motion; the end effecter angle error, has a medium affect to the motion. However, even with the existence of the three classes of manufacturing error, the 7-R single closed loop mechanism can still perform the desired end effecter motion, i.e. translation along a straight line.

One possible application example of this mechanism is automotive suspension as shown in Fig. 12.

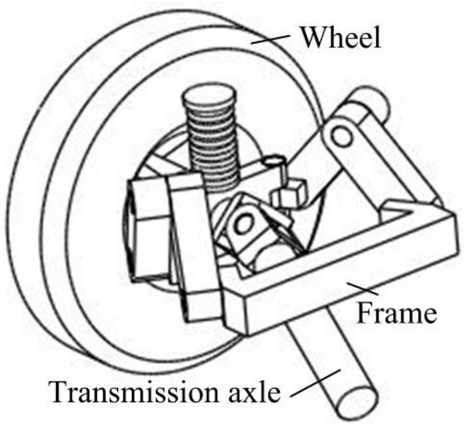

Figure 12. Automotive suspension with 7-R mechanism.

As this mechanism is not sensitive to manufacturing error, there will be no deformation due to assembling, and eventually less stress in the link. Thus such suspension can be more durable.

V. Conclusion

This paper introduces a Sarrus Linkage-like 7-R single closed loop mechanism. The mobility analysis of this mechanism is performed by both group theory based method and reciprocal screw theory. Mobility analysis based on group theory illustrated that this 7-R mechanism, as the Sarrus Linkage, allows full-cycle mobility of 1 degree of freedom. Analysis based on reciprocal screw theory shows that the only displacement allowed by the constraints is translation along one straight line. Further, with the existence of 3 different kinds of manufacturing errors, i.e. link length error, frame angle error and end effecter angle error, this mechanism is still movable. While with the last 2 kinds of errors a Sarrus Linkage can not even be assembled without joint clearance or link deformation. As reciprocal screw theory mobility analysis in this paper does not require special quantity condition of joint directions or axis positions, it can be inferred with manufacturing errors, the mechanism can still realize the same rectilinear motion. Motion simulation is taken to illustrate the mobility of this mechanism. As this mechanism is non-overconstrained, there will be less stress. Thus it is possible to adopt this mechanism to build a more durable automotive suspension. The rationale is that the two sub chains can compensate manufacturing errors for each other. This mechanism is an example of the design of mechanisms that are robust to manufacturing errors.

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